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Numerical methods to value an option including risk aversion with a constant relative risk aversion function

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Abstract

Purpose – This study develops a comprehensive discrete numerical model for option valuation that explicitly incorporates risk preferences, which may deviate from risk neutrality. Unlike the traditional binomial tree models – strictly under the risk-neutral paradigm – our framework embeds a constant relative risk aversion (CRRA) utility specification, capturing heterogeneous attitudes toward risk while preserving the arbitrage-free pricing rule.
Design/methodology/approach – The model extends the multiplicative binomial recombination tree (MBRT) by adjusting key parameters – transition probabilities, growth factors, discount rates and drift/diffusion terms – to reflect the investor's degree of risk aversion. The classical Cox-Ross-Rubinstein binomial tree (CRR) emerges as a special case when risk aversion is set to zero. The methodology remains consistent with geometric Brownian motion (GBM) dynamics and is benchmarked against a modified Monte Carlo simulation to ensure robustness.
Findings – Results show that option values can be consistently derived under both traditional risk-neutral settings and preference-driven settings. Sensitivity analysis highlights the impact of time to maturity, volatility, strike price and the risk-free rate under varying levels of risk aversion.
Research limitations/implications – While this research offers significant theoretical and practical contributions, certain limitations warrant further study. Computational complexity: the CRRA-based valuation method introduces additional numerical challenges, requiring precise calibration and advanced optimization techniques. Dependence on risk aversion estimates: the model assumes that investor risk preferences can be accurately measured and remain stable, which may not always reflect dynamic market conditions. Absence of a closed-form solution: our proposed approach lacks an analytical closed-form solution. Therefore, it is crucial to dedicate efforts to its development.
Practical implications – The integration of CRRA utility functions into derivative valuation represents a key innovation, as it explicitly accounts for investor risk preferences beyond the traditional risk-neutral paradigm. This framework advances the literature on utility-based and nonlinear risk-adjusted pricing by demonstrating how variations in the relative risk aversion (RRA) coefficient shape option values. From a practical perspective, the model offers a flexible tool for portfolio managers, traders and policymakers by aligning valuations with observed market behavior while preserving consistency with classical models under specific conditions. Accurate calibration of risk preferences thus becomes essential for reliable pricing and policy design.
Originality/value – The novelty of this research lies in bridging utility-based preferences with recombining lattice valuation: while prior studies focused exclusively on risk-neutral or arbitrage-based approaches, our model incorporates explicit risk aversion into the numerical structure. By deriving general algebraic expressions and validating the framework through numerical experiments, this study offers a tractable and versatile tool for analyzing option prices under heterogeneous risk attitudes, without losing the analytical clarity of traditional methods.

Keywords Pricing, Binomial trees, Monte Carlo, Numerical methods, Options
Paper type Research article

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1. Introduction

Since the seminal work of Black and Scholes (1973), later extended by Merton (1973), the valuation of European options has relied on the assumption that the underlying asset follows a (GBM). Their framework provided the first closed-form solution and established risk-neutral pricing as a cornerstone of modern financial economics. Building on these theoretical foundations, various numerical estimation methods emerged, such as the finite-difference method, Adomian decomposition method and CRR proposed by Cox *et al.* (1979). Monte Carlo simulation techniques, such as the least squares method (LSM) proposed by Longstaff and Schwartz (2001), have been widely used to value American and Bermudan options with early-exercise features (Gapeev and Li, 2022). As Stentoft (2004) showed, LSM converges to the true expectation but may fail to converge to the real value of the option. Later, Létourneau and Stentoft (2014) modified the ordinary least square (OLS) regression by imposing constraints to mitigate the bias from the number of regressors, although asymptotic convergence was not established. More recently, Fabozzi *et al.* (2017) proposed weighted OLS estimations to address heteroscedasticity. Among these approaches, the CRR method remains the most widely used due to its simplicity, ease of construction and ability to replicate the discrete-time dynamics of the price of the underlying asset while providing tractable derivative valuation (Cox *et al.*, 1979).

Discrete-time extensions have further advanced option valuation. Marín-Sánchez (2010) generalized the CRR through the MBRT method, while Milanesi (2020) incorporated isoelastic utilities and time-varying volatility in a trinomial grid for real options. More recently, Marín-Sánchez *et al.* (2021) employed quadrinomial trees to capture stochastic volatility, reinforcing the flexibility of lattice methods to manage nonlinear dynamics. These contributions highlight the adaptability of discrete models, yet they continue to assume risk-neutral investors.

The persistence of risk neutrality underscores a key gap in the literature: most frameworks do not explicitly account for heterogeneous investor risk preferences, despite empirical evidence that option prices embed valuable information about market risk preferences (Bliss and Panigirtzoglou, 2004). Existing attempts to incorporate preferences often rely on continuous-time models with high computational cost or lack direct adaptation to discrete-time settings.

Our contribution aims to fill this gap by proposing a discrete-time option pricing model that directly incorporates investor risk preferences through a CRRA utility specification. We adapt and extend the MBRT framework to value financial options using certainty-equivalent (CE) approach. Under the CE framework introduced by Beedles (1978), the value of a risky payoff is defined by its utility-adjusted equivalence to a certain outcome. Unlike classical models, our approach does not impose a full risk neutrality but instead reflects the actual risk attitudes of economic agents.

Importantly, while discounting remains risk-neutral, payoffs are adjusted using a CRRA utility function. This separation preserves flexibility and tractability: preferences need only satisfy standard monotonicity (more is preferred to less), while allowing for heterogeneous degrees of risk aversion. In the limiting case where $\gamma = 0$, our framework collapses to the risk-neutral benchmark, recovering classical pricing outcomes in both payoff equivalence and the discounting rule. In this sense, our framework generalizes traditional valuation models – including Black and Scholes (B-S), CRR and LSM – by preserving the risk-neutral discounting of expected payoffs (Björk, 2009), while introducing a nonlinear, preference-sensitive transformation of option values.

The remainder of the paper is organized as follows. Section 2 reviews the theoretical foundation of utility functions, focusing on CRRA and its link to consumption in option pricing. Section 3 derives the main equations, establishes the first two moments of the stochastic process, and details the MBRT framework with supporting algebraic expressions. Section 4 presents numerical experiments analyzing the impact of risk preferences on option values. Section 5 discusses the model's contribution to nonlinear risk-adjusted pricing, its practical relevance and limitations and suggests directions for future research. Section 6 concludes the study by emphasizing the importance of incorporating risk preferences into option valuation to enrich both theory and practice.

2. Literature review

2.1 Expected utility theory, certainty equivalents and utility functions

Decision-making under uncertainty is inherently challenging, requiring methodologies that extend beyond traditional approaches. In academia, risk is commonly modeled using expected utility theory (EUT), which reveals investor preferences by assigning values to the utility generated by alternatives choices, such as investment and consumption. Rooted in neoclassical finance (Shefrin, 2010), EUT follows a rigorous and systematic process that excludes behavioral assumptions. Kahnemann and Tversky (1979) argued that, from the perspective of EUT, decision analysis under risk constitutes a normative model of rational selection, applicable to a wide range of economic behaviors. A detailed review of utility functions and empirical methods for estimating risk aversion is provided by Chávez *et al.* (2017), who highlight key theoretical foundations and estimation techniques applied in financial decision-making contexts.

Risk aversion is generally quantified through two principal measures. The first is absolute risk aversion (ARA), also known as the Arrow–Pratt coefficient, expressed as $r_A(x) = u''(x)/u'(x)$, where $u(x)$ is the utility function, $u'(x)$ is the marginal utility of wealth and $u''(x)$ is its second derivative. To normalize by wealth, the relative risk aversion (RRA) coefficient is defined as $r_R(x) = -x\ u''(x)/u'(x)$.

Consistent with both theory and empirical evidence, we adopt the CRRA utility function to model investor behavior under uncertainty. CRRA satisfies the following key properties: decreasing ARA, constant RRA and scale invariance. It provides analytical while capturing varying degrees of concavity in preferences. In the context of option pricing, CRRA facilitates both the adjustment of investor preferences and correct mathematical manipulation (Bliss and Panigirtzoglou, 2004; Brenner, 2015; Zhou, 2021). Experimental designs based on random lottery pairs further validate its empirical relevance, allowing estimation of individual risk-aversion parameters in controlled environments (Pareja-Vasseur and Baena, 2018).

The CRRA utility function takes the following form:

$$u(x) = \begin{cases} x^{(1-\gamma)}/(1-\gamma). & if \gamma \neq 1 \\ \ln(x) & f\gamma = 1 \end{cases}$$

Here, $x > 0$ represents wealth or a monetary payoff, and $\gamma \in$ is the coefficient of CRRA utility function. This parameter defines attitudes toward risk:

(1) $\gamma > 0$: risk-averse.

(2) $\gamma = 0$: risk-neutral.

(3) $\gamma < 0$: risk-loving.

The function is strictly increasing and concave for $\gamma > 0$, ensuring higher wealth leads to higher utility but with diminishing marginal returns. CRRA preferences are consistent with equilibrium results in the B-S model (Rubinstein, 1976; Brennan, 1979). By contrast, constant absolute risk aversion (CARA) utility is tractable in additive models, it assumes wealth-invariant risk aversion, conflicting with empirical evidence. Epstein–Zin preferences offer greater flexibility by separating risk aversion from intertemporal substitution but introduce significant computational complexity and are beyond the scope of this discrete-time setting. We acknowledge that CRRA does not fully disentangle time and risk preferences; its balance of realism, tractability and empirical validation makes it well-suited for our model. Indeed, CRRA preferences have been successfully applied to dynamic asset–liability management problems under model uncertainty, confirming its tractability and robustness in intertemporal decision-making contexts (Zhao, 2021). Future research may explore Epstein–Zin utility to generalize this framework in dynamic contexts.

Table 1 summarizes empirical estimates of the values of γ reported in the literature.

Table 1 presents a range of empirical RRA coefficients, some outside the domain adopted in our model. Several studies infer investors' risk preferences by contrasting observed market outcomes with risk-neutral benchmarks. For example, Bliss and Panigirtzoglou (2004) derived an adjusted risk-neutral probability density function incorporating a utility function to estimate the RRA from the FTSE 100 and S&P 500 option indices. Similarly, Ait-Sahalia and Lo (2000) introduced the Economic Value-at-Risk (E-VaR), a non-parametric risk measure based on state-price densities that better captures the market risk profile compared to traditional VaR.

While these contributions offer valuable insights, our model constrains the RRA coefficient (γ) to the interval $-1 \leq \gamma < 1$, ensuring that the CRRA utility remains strictly concave, well-defined and computationally stable. We explicitly exclude the boundary case $\gamma = 1$, as it corresponds to a logarithmic utility function, a distinct model with different analytical properties that is not compatible with our current framework. Values outside this interval may result in excessive curvature or undefined CE, compromising both numerical consistency and interpretability. While Table 1 reports broader empirical estimates for completeness, all simulations and calibrations in this study are restricted within the specified domain. This modeling choice aligns with critiques of the risk-neutral valuation paradigm, such as those advanced by Friend (1977), reinforcing the behavioral and structural coherence of our approach.

2.2 Consumption

Pratt (1964) established the foundations for analyzing risk aversion by introducing concepts such as risk premium, decreasing risk aversion and their proportional relation to total assets. Building on this, Fama (1970) showed that under uncertainty, even risk-averse investors maximize lifetime utility, and that nonstationary investment–consumption processes can be represented through non-stochastic single-period utility functions. Extending the utility framework, Richard (1975) proposed a multidimensional definition of RRA, highlighting consumption as a key determinant.

In parallel, Black and Scholes (1973) and Merton (1973) introduced asset pricing methods under a martingale framework, grounded in no-arbitrage and the uniqueness of the risk-neutral measure in incomplete markets. Thijssen (2008) illustrated this within a two-period economy, showing that future prices can be expressed as equivalent martingales and valued relative to present consumption.

Table 1. Estimated values of relative risk aversion (RRA) coefficient

Study	CRRA range
Arrow (1971)	1
Friend (1977)	2
Hansen and Singleton (1982, 1984)	0–1
Mehra and Prescott (1985)	0–10
Normandin and St-Amour (1998)	<3
Coutant (1999)	0–11.404
Ait-Sahalia and Lo (2000)	12.7
Guo and Whitelaw (2006)	3.52
Bliss and Panigirtzoglou (2004)	0.37–15.97
Brenner (2015)	−0.10–6.17
Obrimah (2019)	1.91–3.59
Alexander *et al.* (2021)	1.8–2.1
Laibson *et al.* (2024)	1.9

Source(s): Authors' own elaboration

Empirical evidence was provided by Brenner (2015), who analyzed 65,000 RRA coefficients from 7,000 U.S. executives (1996–2008) and found consistently moderate risk aversion (mean ≈ 1; median $= 3$). His framework-linked risk attitudes to firm characteristics, stock-option exercise behavior, liquidity needs and psychological effects such as overconfidence.

Ellersgaard and Tegnér (2018) integrated these theoretical and empirical strands by applying a martingale approach under the Heston (1993) model. They derived optimal CRRA-based consumption–wealth strategies in a stochastic volatility setting, empirically validated with both stocks and derivatives, including options.

3. Methodology

In this section, we propose a primary framework based on the traditional differential stochastic equation, deriving new trend and diffusion terms that incorporate the CRRA utility function. Assuming that the underlying stochastic process follows a GBM, we employ the CRRA utility function to model the risk preferences of an economic agent, capturing both risk-aversion and risk-seeking behaviors.

3.1 Subjective, risk-neutral and risk-preferences

3.1.1 From a subjective agent-based world to a neutral risk-decision world. In a risk-neutral setting, two critical conditions are essential for option valuation. First, estimated future cash flows are determined by discounting their expected values at the risk-free interest rate. A single neutral probability measure (P^*) must be derived from the subjective probability (P) to prevent arbitrage on the discounted expected value. This ensures that the discounted price at an interest rate is a martingale and that the discounted expected value under this rate, according to the new probability (P^*), does not present arbitrage opportunities. The probability measure used in a risk-neutral world is the martingale equivalent measure (MEM).

Consider the following traditional linear homogeneous differential equation [1]:

$$dS_t = \mu S_t dt + \sigma S_t dB_t \tag{1}$$

That describes the behavior of an specific underlying asset in a specific market, where $S_0 = S$; μ is a constant that denotes the asset's average rate return, $\sigma > 0$ is the annual volatility, and $\{B_t\}_{t \geq 0}$ is a standard one-dimensional Brownian motion (SODBM) define in the probability $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, P)$.

Consider the discounted pay based on the risk-free rate r as $S_t^* = e^{-rt} S_t$. Then, apply Ito's lemma.

$$dS_t^* = e^{-rt}(-r)S_t dt + e^{-rt} dS_t + \frac{1}{2}(0)$$

Allow us to get:

$$dS_t^* = (\mu - r)S_t^* dt + \sigma S_t^* dB_t \tag{2}$$

This process is supported by the theorem introduced by Heston (1993) and the application of the Cameron–Martin–Girsanov theorem as well as Girsanov's theorem (Mao, 1997). Then,

$$\phi(t) = -\frac{(\mu - r)S_t^*}{\sigma S_t^*} \quad \phi(t) = -\frac{(\mu - r)}{\sigma} \tag{3}$$

It is clear that $\phi(t) \in L^2([0,T]; \quad)$. Then, we obtain

$$dB_t^* = dB_t - \phi(t)\, dt \qquad (4)$$

Substituting (3) into (4)

$$dB_t = dB_t^* - \frac{(\mu - r)}{\sigma} dt \qquad (5)$$

Substituting (5) into (2), we got

$$dS_t^* = (\mu - r)S_t^* dt + \sigma S_t^* \left(dB_t^* - \frac{(\mu - r)}{\sigma} dt \right)$$

$$dS_t^* = \sigma S_t^* dB_t \quad S_t^* = S_t^* + \int_0^t S_t^* dB_s$$

It is worth to mention that S_t^* is Martingale respect $\mathcal{F}_t$ to $0 \leq \tau \leq t$. Now, replace (5) in (1).

$$dS_t = rS_t dt + \sigma S_t dB_t^* \qquad (6)$$

Note that the previous equation corresponds to the behavior of an underlying price in a risk-neutral world.

3.1.2 From a neutral risk-decision world to a world of risk-preferences. Consider the traditional neutralized linear homogeneous differential equation (6) which describes the behavior of an underlying asset on a neutral-risk valuation, where $S_0 = S$; r is the risk-free rate, $\sigma > 0$ is the annual volatility and $\{B_t\}_{t \geq 0}$ is a SODBM defined in the probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t \geq 0}, P)$.

Assume the CRRA utility function $u(x) = \frac{x^{(1-\gamma)}}{(1-\gamma)}$; with $\gamma \neq 1$ and $-1 \leq \gamma < 1$, considering this transformation $Y_t = \frac{S_t^{(1-\gamma)}}{(1-\gamma)}$ and applying Ito's formula to Y_t, we obtain [2]:

$$dY_t = (1-\gamma)\frac{S_t^{1-\gamma-1}}{1-\gamma} dS_t + \frac{1}{2} S_t^{-\gamma-1}(-\gamma)(dS_t)^2$$

$$= \left(r - \frac{\gamma}{2}\sigma^2 \right) S_t^{1-\gamma} dt + \sigma S_t^{1-\gamma} dB_t$$

$$dY_t = \alpha Y_t dt + \theta Y_t dB_t$$

with $\alpha = (1-\gamma)\left(r - \frac{\gamma}{2}\sigma^2\right)$; $\theta = \sigma(1-\gamma)$.

We recognize that incorporating risk aversion into a risk-neutral valuation framework may initially seem contradictory. However, our model resolves this by clearly distinguishing two separate valuation stages.

First, future stochastic payoffs are evaluated in a "risky world" or "risk-adjusted world" (Y), using the CRRA utility function to explicitly incorporate investor risk preferences. This process produces a risk-adjusted rate α, which directly reflects the investor's degree of risk aversion. When the risk aversion coefficient $\gamma = 0$, α coincides exactly with the risk-free rate, thereby recovering the standard risk-neutral benchmark.

Second, the adjusted payoffs are discounted to present value in a "risk-neutral world" (S), using the risk-free rate. Therefore, our approach accounts for risk in the payoffs themselves rather than in the discounting process, as summarized by the following formula:

$$F(S) = e^{-r\ T} E[\Phi[S]] \tag{7}$$

here r represents the discount rate and $\Phi(S)$ the final payoff.

This two-step valuation approach resolves the apparent contradiction by clearly separating the incorporation of risk preferences from the no-arbitration discounting condition. As a result, our model can be viewed as a generalization of traditional risk-neutral methods (e.g. B-S, CRR and LSM methods), which discount the expected weighted average payoffs directly under the risk-neutral measure (Björk, 2009).

3.2 Multiplicative binomial tree recombination

To build a multiplicative tree that allows us to describe the discrete behavior of the underlying asset, we must determine the first two moments of the ordinary linear stochastic differential equation to obtain the transition probability, which we present next. These expressions have been widely employed to calibrate recombining lattice models (Lari-Lavassani *et al.*, 2001; Marín-Sánchez, 2010), which we adopt in this work [3].

Proposition 1. Consider the following stochastic differential equation

$$dY_t = \alpha Y_t dt + \theta Y_t dB_t \tag{8}$$

on the time interval $[t_i, t_k]$, where $\alpha \in \ $ and $\theta > 0$ are constants, and $\{B_t\}_{t \geq 0}$ is a SODBM defined in the probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t \geq 0}, P\ )$. Additionally, we assume that $Y_t = Y_i$.

(1) *The first two moments are calculated as*

- $E[Y_t | Y_{t_i}] = Y_{ti} \exp(\alpha(t - t_i))$
- $E[Y_t^2 | Y_{t_i}] = Y_{t_i}^2 exp((2\alpha + \theta^2)(t - t_i))$

Proof 1. Equation (8) can be expressed in an integral form as

$$Y_t - Y_{t_i} = \alpha \int_{t_i}^{t} Y_s ds + \int_{t_i}^{t} Y_s dB_s$$

Now, we take the expected value $\ [\ ]$ that corresponds to the conditional expected value, e.g. $\ [Y_t] = E[Y_t | Y_{t_i}]$.

$$\ [Y_t] - \ [Y_{t_i}] = \alpha \int_{t_i}^{t} \ [Y_s] ds + \theta \ \left[\int_{t_i}^{t} Y_s dB_s \right]$$

Now, with a change in variable $m(t) = \ [Y_t]$,

$$m(t) = m(t_i) + \alpha \int_{t_i}^{t} m[s] ds$$

Thus, we obtain the ordinary differential equation $\dot{m}(t) = \alpha m(t)$. Its solution is given by $m(t) = m(t_i) \exp(\alpha(t_k - t_i))$, hence

$$m(t) \rfloor_{t_k} = Y_{t_i} \exp(\alpha(t_k - t_i)) = Y_{t_i} \exp(\alpha \Delta t) \tag{9}$$

(2) Let $Z_t = Y_t^2$; applying the Ito's formula to Y_t, we obtain

$$\begin{aligned} dZ_t &= 2Y_t dY_t + \frac{1}{2}(2)(dY_t)^2 \\ &= \left(2\alpha + \theta^2\right) Y_t^2 dt + 2\theta Y_t^2 dB_t \end{aligned} \tag{10}$$

In an integral form, Equation (10) becomes

$$Y_t^2 - Y_{t_i}^2 = \left(2\alpha + \theta^2\right) \int_{t_i}^{t} Y_s^2 ds + 2\theta \int_{t_i}^{t} Y_s^2 dB_s$$

We take the expected value of both sides

$$E\left[Y_t^2\right] - E\left[Y_{t_i}^2\right] = \left(2\alpha + \theta^2\right) \int_{t_i}^{t} E\left[Y_s^2\right] ds + 2\theta \left[\int_{t_i}^{t} Y_s^2 dB_s\right]$$

We change variables $P(t) = \quad [Y_t^2]$, so

$$P(t) = P(t_i) + \left(2\alpha + \theta^2\right) \int_{t_i}^{t} P(s) ds$$

This way, the ordinary differential equation $P(t) = (2\alpha + \theta^2)P(t)$; its solution is given by

$$P(t) = P(t_i)\exp\left(\left(2\alpha + \theta^2\right)(t - t_i)\right)$$

Therefore

$$\left[Y_t^2\right] = Y_t^2 \exp\left(\left(2\alpha + \theta^2\right)(t_k - t_i)\right) = Y_t^2 \exp\left(2\alpha + \theta^2\right)\Delta t \tag{11}$$

3.3 Calculation of transition probability, growth factor and discount rate to Y_t

Proposition 2. Consider the stochastic differential Equation (8) *and the MBRT $Y^{(i)} = Y_j^{(i)}$. Assuming that over the time interval $[t_i, t_i + \Delta t]$, we define the conditional expectation of the neutralized continuous process as $E[Y_t|\ \mathrm{Y}_{t_i}]\big|_{t_i+\Delta t}$ and for the discrete process as $E_d\left[Y^{(i)}|\ Y_j^{(i)}\right]\Big|_{t_k t_i+\Delta t}$.*

The match between the first and second moments for both processes allow us to obtain a specific recombination given by

$$p_j^{(i)} = \frac{A(Y_{t_i}, \Delta t) - d_j^{(i)}}{u_j^{(i)} - d_j^{(i)}}; \quad u_j^{(i)} = \exp\left(\cosh^{-1}(C(Y_{t_i}, \Delta t))\right); \quad d_j^{(i)} = \frac{1}{u_j^{(i)}}.$$

where $A(Y_{t_i}, \Delta t) := \frac{E\left[Y_t|Y_{t_i}\right]\big|_{t_i+\Delta t}}{Y_j^{(i)}}$; $B(Y_{t_i}, \Delta t) := \frac{E\left[Y_t^2|Y_{t_i}\right]\big|_{t_i+\Delta t}}{\left(Y_j^{(i)}\right)^2}$; $C(Y_{t_i}, \Delta t) := \frac{1+B\left(Y_{t_i},\Delta t\right)}{2A\left(Y_{t_i},\Delta t\right)}$

Proof. Using the definition of $E_d[\ |\ ]$, the matching between the discrete and continuous time processes is

$$E_d\left[Y^{(i)}|\ Y_j^{(i)}\right]\rfloor_{t_i+\Delta t} := p_j^{(i)}Y_{j+1}^{(i+1)} + \left(1-p_j^{(i)}\right)Y_j^{(i+1)} = E[Y_t|\ Y_{t_i}]\rfloor_{t_i+\Delta t}$$

$$E_d\left[\left(Y^{(i)}\right)^2|\ Y_j^{(i)}\right]\rfloor_{t_i+\Delta t} := p_j^{(i)}\left(Y_{j+1}^{(i+1)}\right)^2 + \left(1-p_j^{(i)}\right)\left(Y_j^{(i+1)}\right)^2 = E\left[Y_t^2|\ Y_{t_i}\right]\rfloor_{t_i+\Delta t}$$

Because $Y_{j+1}^{(i+1)} = Y_j^{(i)}u_j^{(i)}$ and $Y_j^{(i+1)} = Y_j^{(i)}d_j^{(i)}$ the previous equations can be reduced to

$$p_j^{(i)}Y_j^{(i)}u_j^{(i)} + \left(1-p_j^{(i)}\right)Y_j^{(i)}d_j^{(i)} = \ [Y_t]\rfloor_{t_i+\Delta t}$$

$$p_j^{(i)}\left(Y_j^{(i)}u_j^{(i)}\right)^2 + \left(1-p_j^{(i)}\right)\left(Y_j^{(i)}d_j^{(i)}\right)^2 = \ \left[Y_t^2\right]\rfloor_{t_i+\Delta t}$$

The previous equations can be rewritten as

$$p_j^{(i)}u_j^{(i)} + \left(1-p_j^{(i)}\right)d_j^{(i)} = \frac{[Y_t]\rfloor_{t_i+\Delta t}}{Y_j^{(i)}} := A(Y_{t_i}, \Delta t) \quad (12)$$

$$p_j^{(i)}\left(u_j^{(i)}\right)^2 + \left(1-p_j^{(i)}\right)\left(d_j^{(i)}\right)^2 = \frac{\left[Y_t^2\right]\rfloor_{t_i+\Delta t}}{\left(Y_j^{(i)}\right)^2} := B(Y_{t_i}, \Delta t) \quad (13)$$

From Equation (12) we obtain:

$$p_j^{(i)} = \frac{A(Y_{t_i}, \Delta t) - d_j^{(i)}}{u_j^{(i)} - d_j^{(i)}} \quad (14)$$

Substituting Equation (14) into Equation (13) we obtain:

$$B(Y_{t_i}, \Delta t) = A(Y_{t_i}, \Delta t)\left(u_j^{(i)} + d_j^{(i)}\right) - u_j^{(i)}d_j^{(i)} \quad (15)$$

Without loss of generality and to eliminate a degree of freedom, we assume $u_j^{(i)}d_j^{(i)} = 1$ therefore, from Equation (15) we obtain

$$\frac{u_j^{(i)} + d_j^{(i)}}{2} = \frac{1 + B(Y_{t_i}, \Delta t)}{2A(Y_{t_i}, \Delta t)} := C(Y_{t_i}, \Delta t)$$

If we assume that, based on $C(Y_{t_i}, \Delta t)$ we can derive an expression to determine $u_j^{(i)}$, we obtain the following expression:

$$\ln\left(u_j^{(i)}\right) = \ln\left(C(Y_{t_i}, \Delta t) + \sqrt{C^2(Y_{t_i}, \Delta t) - 1}\right)$$

And finally

$$u_j^{(i)} \quad = \quad \exp\left(\cosh^{-1}(C(Y_{t_i}, \Delta t))\right) \quad (16)$$

Proposition 3. Consider the stochastic differential Equation (8) on the time interval $[t_0, T]$*. Let* $Y^{(i)} = (Y_j^{(i)})$ *a recombination based on a MBRT with time step* Δt*. Assume that there is a low boundary* $Y_{\min} > 0$ *and* $Y_{\min} \leq Y_j^{(i)}$ *for all i and j. If we match the first and second moments for the discrete and continuous processes respectively, we get a complete recombination given by*

$$u_j^{(i)} = \exp\left(\theta\sqrt{\Delta t}\right), d_j^{(i)} = \frac{1}{u_j^{(i)}}, A = \exp(\alpha\Delta t) \text{ and } p_j^{(i)} = \frac{1}{2} + \left(\frac{\alpha - \frac{1}{2}\theta^2}{2\theta}\right)\sqrt{\Delta t}$$

Proof. Starting with Taylor's series expansion with respect to $Y^{(i)}$ and centered on Δt. From Proposition 2 and Equation (9), we find

$$A(Y_{t_i}, \Delta t) = \frac{Y_{t_i}\exp(\alpha\Delta t)}{Y_j^{(i)}} = \exp(\alpha\Delta t) \tag{17}$$

Similarly, using Equation (11), we find

$$B(Y_{t_i}, \Delta t) = \frac{Y_{t_i}^2\exp\left(2\alpha + \theta^2\right)\Delta t}{\left(Y_j^{(i)}\right)^2} = \exp\left(2\alpha + \theta^2\right)\Delta t$$

Hence

$$C(Y_{t_i}, \Delta t) = \frac{1 + B(Y_{t_i}, \Delta t)}{2A(Y_{t_i}, \Delta t)} = \frac{2 + \theta^2\Delta t + O(\Delta t)}{2} \tag{18}$$

So,

$$C^2(Y_{t_i}, \Delta t) = \frac{4 + 4\theta^2\Delta t + O(\Delta t)}{4} \tag{19}$$

Substituting Equations (18) and (19) on (16), and taking the positive solution of the square root, we obtain

$$\begin{aligned} u_j^{(i)} &= \frac{2 + \theta^2\Delta t + O(\Delta t)}{2} + \sqrt{\frac{4 + 4\theta^2\Delta t + O(\Delta t)}{4} - 1} \\ u_j^{(i)} &= \exp\left(\theta\sqrt{\Delta t}\right) = \frac{1}{d_j^{(i)}} \end{aligned} \tag{20}$$

Finally, we calculate $p_j^{(i)}$ from Equations (17) and (20), which are substituted for Equation (14) as follows

$$p_j^{(i)} = \frac{A(Y_{t_i}, \Delta t) - d_j^{(i)}}{u_j^{(i)} - d_j^{(i)}}$$

$$p_j^{(i)} = \frac{1}{2} + \frac{\left(\alpha - \frac{1}{2}\theta^2\right)\sqrt{\Delta t}}{2\theta} + O(\Delta t)$$

3.4 Multiplicative binomial recombination tree

Consider a stochastic process S^t modeled discreetly over time. In each time period i, the possible states of the process are given by a finite number of vectors $S^{(i)} = (S_j^{(i)})$, where j is a finite indexed set $J^{(i)}$. Just as the time stream runs from i to period $i+1$, discrete node $S_j^{(i)}$ can become part of any $S_j^{(i+1)}, j \in J^{(i+1)}$.

The discrete process determines a transition probability matrix $P^{(i)} = P_{j,j}^{(i)}$, and thus the element $P_{j,j}^{(i)}$ represents the probability that $S_j^{(i)}$ moves to $S_j^{(i+1)}$. The number of rows in $P^{(i)}$ is the cardinal of $J^{(i)}$ and the number of columns is the cardinal of $J^{(i+1)}$, such that all $j \in J^{(i)}$ and $\sum_{j \in J^{(i+1)}} P_{j,j}^{(i)} = 1$.

The transition probabilities are given for each i by a vector $P_j^{(i)}, j \in J^{(i)}$ such that $P_{i,j}^{(i)}$ is defined by $P_j^{(i)}$ if $J = j+1, 1 + P_j^{(i)}$ if $J = j$, and zero otherwise.

In this multiplicative process, the size of the upward and downward jumps is defined by $u_j^{(i)}$ and $d_j^{(i)}$, respectively, so that $S_{j+1}^{(i+1)} = S_j^{(i)} u_j^{(i)}$ and $S_j^{(i+1)} = S_j^{(i)} d_j^{(i)}$ such that $u_j^{(i)} d_{j+1}^{(i+1)} = d_j^{(i)} u_j^{(i+1)}$; obtaining the recombination given by

$$\begin{cases} S_j^{(i)} = S_j^{(i)} \prod_{k=1}^{j-1} u_k^{(k-1)} \prod_{k=1}^{i-j+1} d_k^{(k-1)}, j = 1, 2 \ldots, i+1 \\ S_0 = S_1^{(0)} \end{cases} \tag{21}$$

Note that in the special case where $u_j^{(i)} = u$ and $d_j^{(i)} = d$ are constants, this recombination takes the form $S_j^{(i)} = S_0 u^{j-1} d^{i-j+1}$.

3.5 Option valuation

In this section, we provide a brief overview of the MBRT with CRRA (MBRT–CRRA) method for option valuation, incorporating a CRRA-type utility function that corresponds to the subjective probability P. This probability is defined in the real-world measure, accounting for arbitrage and the absence of risk neutrality. Finally, we present the necessary transformation that enables evaluation under the risk-neutral measure.

Assume that the financial option of an asset with an initial value of $Y_0 = \frac{S_0^{(1-\gamma)}}{1-\gamma}$ and exercise price of K, and T is divided into N subintervals, each with a length of Δt. We define $f_j^{(i)}$ as the value of the option in the (i, j) node. Based on Marín-Sánchez (2010), the price of the asset, expressed as a general function, has binomial recombination in the (i, j) node, which can be represented by the following equation:

$$Y_j^{(i)} = Y_0 \prod_{k=1}^{j-1} u_k^{(k)} \prod_{k=1}^{i-j} d_k^{(i-k)} \tag{22}$$

with $Y_0 = Y_1^{(1)}$; $i = 0, 1, \ldots, N$; $j = 1, 2, \ldots, i$. It is important to note that, in this case, both u and d are constants Lari-Lavassani *et al.* (2001), so Equation (22) is summarized as follows:

$$Y_j^{(i)} = Y_0 u^{j-1} d^{i-j}$$

3.5.1 Financial options. This is followed by an algebraic approach derived from the proposed methodology to assess the basic financial options.

In case of a European call option, the value on the maturity date is indicated by $\frac{(\max(S_t - K, 0))^{1-\gamma}}{1-\gamma}$, where $S_t = ((1-\gamma)Y_t)^{\frac{1}{1-\gamma}}$, recall that $\gamma \neq 1$ and $-1 \leq \gamma < 1$, so

$$f_j^{(N)} = \frac{\left\{\max\left(\left[(1-\gamma)Y_0 u^{j-1} d^{N-j}\right]^{\frac{1}{1-\gamma}} - K, 0\right)\right\}^{1-\gamma}}{1-\gamma}$$

whereas the value at each node is given by

$$f_j^{(i)} = \left(p_j^{(i)} f_{j+1}^{(i+1)} + \left(1 - p_j^{(i)}\right) f_j^{(i+1)}\right) A^{-1} \tag{23}$$

The same analysis can be applied to valuing European put options. In the case of an American call option, the value at maturity is estimated as the value of a European call option, while the discounted value is defined by

$$f_j^{(i)} = \max\left(\frac{\left\{\left(\left[(1-\gamma)Y_0 u^{j-1} d^{i-j}\right]^{\frac{1}{1-\gamma}} - K\right)\right\}^{1-\gamma}}{1-\gamma}, \left(p_j^{(i)} f_{j+1}^{(i+1)} + \left(1 - p_j^{(i)}\right) f_j^{(i+1)}\right) A^{-1}\right) \tag{24}$$

Similarly, we can represent the value of the maturity date for an American put option. In the case of a Bermudan call option, the value at maturity is estimated in the same way as for an American or European option, but in some switching periods, $f_j^{(i)}$ changes from Equation (24) to Equation (23).

3.6 Numerical study

To ensure that the option valuation does not yield incorrect or spurious solutions at the nodes of the binomial tree, it is essential to verify the numerical properties of the proposed schemes. For instance, we verify whether the probability of transition lies within the interval $[0, 1]$, whether the binomial recombination representing prices remain non-negative, and whether the valuation scheme is positive a monotonically increasing in the time direction. Once these conditions are identified, we establish specific constraints for the scheme that adhere to ideal properties, which are maintained through a series of propositions proven in the next section.

3.6.1 Constrained probability.

Proposition 4. *We consider the transition probability $p_j^{(i)}$ specified in Proposition 3.*

If $\left(\frac{\alpha - \frac{1}{2}\theta^2}{\theta}\right) T \leq N$, then

$$0 \le p_j^{(i)} \le \forall_{i,j};\ i = 0, 1, 2, \ldots, N\ and\ j = 0, 1, 2, \ldots, i-1$$

Proof. Consider that

$$\left(\frac{\alpha - \frac{1}{2}\theta^2}{\theta}\right)^2 \left(\sqrt{T}\right)^2 \le \left(\sqrt{N}\right)^2 \text{ Therefore, } \sqrt{\left(\frac{\alpha - \frac{1}{2}\theta^2}{\theta}\right)^2}\sqrt{\left(\sqrt{T}\right)^2} \le \sqrt{\left(\sqrt{N}\right)^2}$$

$\left|\frac{\alpha - \frac{1}{2}\theta^2}{\theta}\sqrt{\Delta t}\right| \le 1$ and finally, $0 \le \frac{1}{2} + \frac{1}{2}\left(\frac{\alpha - \frac{1}{2}\theta^2}{\theta}\right)\sqrt{\Delta t} \le 1$ providing that $0 \le p_j^{(i)} \le 1$.

3.6.2 Positivity.

Proposition 5. *Under the same assumptions as in Proposition 4, the MBRT–CRRA scheme presented in Section 3.5.1 is positive for $i = 0, 1, 2, \ldots, N$ and $j = 0, 1, 2, \ldots, i-1$.*

Proof. Note that $f_j^{(N-1)} = (p_j^{(N-1)} f_{j+1}^{(N)} + (1 - p_j^{(N-1)}) f_j^{(N)}) A^{-1}$. With non-negative payoffs $f_j^{(N)}, f_{j+1}^{(N)}$ and $0 \le p_j^{(N-1)} \le 1$, then $f_j^{(N-1)} \ge 0$ for $j = 0, 1, 2, \ldots, N-2$. Now, based on an inductive hypothesis, suppose that $f_j^{(N-K)} \ge 0$ for $j = 0, 1, 2, \ldots, N-K-1$. Following the previous step, we found that $f_j^{(N-K-1)} = [p_j^{(N-K-1)} f_{j+1}^{(N-K)} + (1 - p_j^{(N-K-1)}) f_j^{(N-K)}] A^{-1}$.

Because $p_j^{(N-K-1)} \ge 0$ and considering the inductive process, we obtain $f_j^{(N-K-1)} \ge 0$. Thus, $f_j^{(i)} \ge 0$ for $i = 0, 1, 2, \ldots, N$ and $j = 0, 1, 2, \ldots, i-1$.

Remark 1. *It is relatively simple to check the positivity of MBRT–CRRA Equation (22) for $Y_0 > 0$, $u_j^{(i)} > 0$ and $d_j^{(i)} > 0$.*

3.6.3 Monotonicity.

Definition 1. *Consider the MBRT–CRRA scheme in Equation (23). Thus, it is monotonically* conservative scheme if we assume that $f_{j+1}^{(i+1)} - f_j^{(i+1)} \ge 0$ and $f_j^{(i+1)} - f_{j-1}^{(i+1)} \ge 0$ Then, $f_j^{(i)} - f_{j-1}^{(i)} \ge 0\ \forall_i \in I$ and $\forall_j \in J$, where I and J are non-negative integers.

Proposition 6. *Under the same assumptions of Proposition 4, the MBRT–CRRA scheme presented in Equation (23) for a European call option is monotonically conservative for $1 \le i \le N$ and $1 \le j \le i$*

Proof. For $0 \le p_{j-1}^{(i)} \le 1$, we find that $-1 \le p_{j-1}^{(i)} - 1 \le 0$; because $f_j^{(i)} - f_{j-1}^{(i)} \ge 0$, then

$$-\left[f_j^{(i)} - f_{j-1}^{(i)}\right] \le \left[p_{j-1}^{(i)} - 1\right]\left[f_j^{(i)} - f_{j-1}^{(i)}\right] \le 0 \quad (25)$$

In the same way, $0 \le p_{j-1}^{(i)} \le 1$, so

$$0 \le p_j^{(i)}\left[f_j^{(i)} - f_{j-1}^{(i)}\right] \le \left[f_j^{(i)} - f_{j-1}^{(i)}\right] \quad (26)$$

From Equations (25) and (26) we obtain

$$\left[p_{j-1}^{(i)}-1\right]\left[f_j^{(i+1)}-f_{j-1}^{(i+1)}\right]\leq p_j^{(i)}\left[f_{j+1}^{(i+1)}-f_j^{(i+1)}\right]$$

$$p_{j-1}^{(i)}f_j^{(i+1)}+\left[1-p_{j-1}^{(i)}\right]f_{j-1}^{(i+1)}\leq p_j^{(i)}f_{j+1}^{(i+1)}+\left[1-p_{j-1}^{(i)}\right]f_j^{(i+1)}$$

$$f_j^{(i)}-f_{j-1}^{(i)}\geq 0$$

Lemma 1. The MBRT–CRRA Equation (22) is i-monotonically decreasing and j-monotonically increasing, that is, $Y_j^{(i)}\leq Y_j^{(i-1)}$ and $Y_j^{(i)}\geq Y_{j-1}^{(i)}\ \forall_i;\ i=1,2,\ldots,N$ and $j=1,2,\ldots,i$.

Proof. For $d=e^{(-\theta\sqrt{\Delta t})}$ we have $1\geq d\geq 0\rightarrow 1\geq d^2$ therefore, $d=\frac{1}{u}$, and $1\geq u^{-1}d$. Now consider

$$d^{i-j}\geq u^{-1}d^{i-j+1}\rightarrow u^{j-1}d^{i-j}\geq u^{j-2}d^{i-(j-1)}$$

Then

$$Y_0u^{j-1}d^{i-j}\geq Y_0u^{j-2}d^{i-(j-1)}\rightarrow Y_j^{(i)}\geq Y_{j-1}^{(i)}$$

Similarly, we can prove that $Y_j^{(i)}\geq Y_j^{i-1}$ begins with $1\leq u$.

Proposition 7. The MBRT–CRRA Equation (24) to value American call options, is monotonically conservative for $1\leq i\leq N$ *and* $1\leq j\leq i$.

Proof. Based on Lemma 1 and Remark 1, we have $Y_j^{(i)}\geq Y_{j-1}^{(i)}\geq 0$. For $1-\gamma>0$

$$(1-\gamma)Y_j^{(i)}\geq(1-\gamma)Y_{j-1}^{(i)}$$

And due to $\frac{1}{1-\gamma}>0$, we obtained

$$\left[(1-\gamma)Y_j^{(i)}\right]^{\frac{1}{1-\gamma}}-K\geq\left[(1-\gamma)Y_{j-1}^{(i)}\right]^{\frac{1}{1-\gamma}}-K$$

So

$$\frac{\left\{\left[(1-\gamma)Y_j^{(i)}\right]^{\frac{1}{1-\gamma}}-K\right\}^{1-\gamma}}{1-\gamma}\geq\frac{\left\{\left[(1-\gamma)Y_{j-1}^{(i)}\right]^{\frac{1}{1-\gamma}}-K\right\}^{1-\gamma}}{1-\gamma}\tag{27}$$

For simplicity, let us examine the following variable changes:

$$w_j^{(i)}=\frac{\left\{\left[(1-\gamma)Y_j^{(i)}\right]^{\frac{1}{1-\gamma}}-K\right\}^{1-\gamma}}{1-\gamma};\ \widetilde{f}_j^{(i)}=\left\{p_j^{(i)}f_{j+1}^{(i+1)}+\left(1-p_{j-1}^{(i)}\right)f_j^{(i+1)}\right\}A^{-1}$$

By applying Definition 1 and following calculations similar to those in Proposition 6, it is easy to establish that

$$\widetilde{f}_j^{(i)} \geq \widetilde{f}_{j-1}^{(i)} \tag{28}$$

With that in mind, Equation (24) could be rewritten as $f_j^{(i)} = max\{w_j^{(i)}, \widetilde{f}_j^{(i)}\}$. In addition, in Equation (27) note that $w_j^{(i)} \geq w_{j-1}^{(i)}$, so finally, we can conclude that $max\{w_j^{(i)}, \widetilde{f}_j^{(i)}\} \geq max\{w_{j-1}^{(i)}, \widetilde{f}_{j-1}^{(i)}\}$ and $f_j^{(i)} - f_{j-1}^{(i)} \geq 0$.

4. Results

4.1 Numerical experiments

4.1.1 Examples of financial options. Suppose a Bermudan call option at different maturities, where the transition between the European and American payoff occurs at $T/2$. Let us assume that the dynamic behavior of the of a given financial asset follow a GBM, as described in Equation (8), with the initial conditions $S_0 = 10$, $K = 10$, $\gamma = 0$, $r = 0.05$, and annualized volatility of $\sigma = 0.30$. These values allow us to calculate the parameters α and θ. In this framework, we consider that investors exhibit either risk-averse or risk-loving behaviors, which are captured by the RRA coefficients $\gamma = 0.3$ and $\gamma = -0.3$ respectively. For reference, we also present the classical risk-neutral case. To evaluate the performance of our approach, we compare the experimental numerical results obtained using the MBRT–CRRA and LSM Monte Carlo Simulation with CRRA (LSM–CRRA [4]) methods. In the LSM–CRRA method, the option value is estimated based on 10,000 simulated trajectories and 1,000 repetitions.

Table 2 shows that option values rise with longer maturities, and both methods yield consistent results, supporting the validity of our framework. Importantly, risk preferences alter premiums: under risk aversion, values are lower than in the risk-neutral case, reflecting the tendency of investors to exercise early to avoid uncertainty. By contrast, risk-loving investors hold positions longer, increasing counterparty exposure and leading to higher premiums relative to the neutral benchmark.

4.2 Sensibility of α parameter and utility respect variations of γ

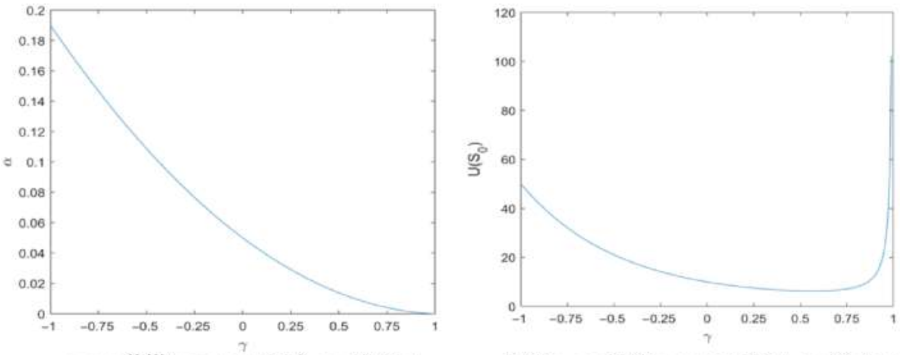
Using the initial conditions of $S_0 = 10$, $N = 100$, $\sigma = 0.3$ and $r = 0.05$ in Equation (8), Figure 1a illustrates that the value of α tends toward zero as coefficient γ approaches $+1$, but remains positive if γ differs from $+1$. Under the B-S valuation, this is exactly 5%, consistent with the assumed risk-free interest rate and corresponds to α is zero meaning that the investor exhibits risk neutrality.

Figure 1b further illustrates that changes in the RRA coefficient directly affect profit price fluctuations. For positive values up to 0.57, profits decline and may even turn negative; beyond this threshold, they rise again, implying that the model should be constrained within this range to maintain economic coherence.

Table 2. Comparison of the value of a Bermudan call option between the MBRT–CRRA and the LSM–CRRA

Case	Method \\T	1	1//2	1//3	1//4	1//6	1//8	1//12
Risk-Averse (gamma = 0.3)	MBRT-CRRA	0.981	0.669	0.538	0.461	0.372	0.32	0.259
	LSM-CRRA	0.997	0.671	0.541	0.466	0.376	0.322	0.261
Risk-Neutral (gamma = 0.0)	MBRT-CRRA	1.422	0.963	0.769	0.658	0.528	0.453	0.366
	LSM-CRRA	1.428	0.963	0.78	0.656	0.534	0.463	0.368
Risk-Lover (gamma = −0.3)	MBRT-CRRA	1.809	1.227	0.981	0.838	0.673	0.577	0.465
	LSM-CRRA	1.823	1.239	0.982	0.842	0.676	0.579	0.467

Source(s): Authors' own elaboration



α sensibility respect RRA co-efficient
(a)

Utility sensibility respect RRA coefficient
(b)

Figure 1. Sensibility analysis respect RRA coefficient $-1 \leq \gamma \leq +1$ with N = 100. **Source(s):** Authors' own elaboration

By contrast, negative values of the coefficient (risk-seeking behavior) lead to steadily increasing profits as risk preference intensifies. It is important to note that when $\gamma = 1$, a discontinuity arises. This issue can be easily resolved using L'Hôpital's rule by transforming the function into a logarithmic form.

4.3 Behavior of S_t and Y_t

Figures 2a and b illustrate the evolution of both S_t and Y_t over time, assuming variations in the parameter α. Figure 2a demonstrates that the price of the underlying asset and its corresponding utility exhibit a similar patterns. The key distinction is that when the utility coefficient is negative, the value of the utility is higher; conversely, when the RRA coefficient is positive, the utility decreases. Nonetheless, the relative changes remain proportional.

4.4 Comparison between MBRT–CRRA, LSM–CRRA and B-S model

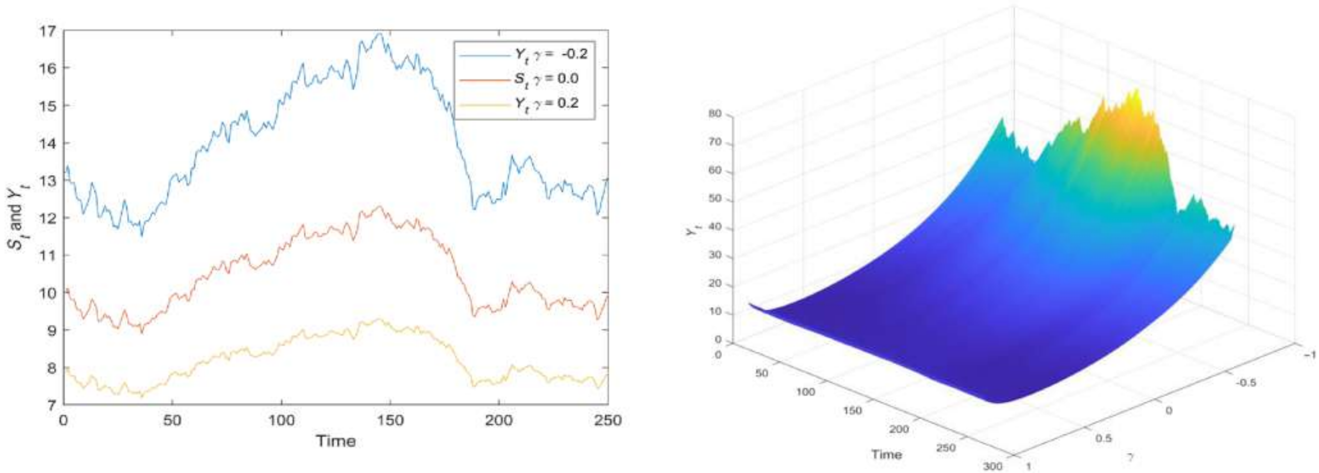
Figure 3a presents the results obtained using the LSM–CRRA and MBRT–CRRA methods, illustrating the convergence between our proposed approach and the B-S methodology for valuing a European call option. The valuation is performed under the initial conditions of $S_0 = 10, K = 10, N = 100, \sigma = 0.3, r = 0.05, T = 1$ years, $\gamma = 0$ and 50 repetitions of 10,000 trajectories.

The MBRT–CRRA method converges rapidly to the exact solution within 40–50 periods, while the LSM–CRRA approximation oscillates below the benchmark and stabilizes after 60 periods.

The numerical results show that the MBRT–CRRA provides an adequate price compared to the LSM–CRRA method, as the difference between the two methods is not significant. Additionally, in terms of time efficiency, the MBRT–CRRA shows an outstanding performance with reductions between 96 and 99% in price estimation. In terms of computational performance, the MBRT–CRRA method requires approximately one second to compute option price with N = 300, by contrast, the LSM–CRRA method takes about three seconds for a single run (10,000 trajectories) and, given its reliance on simulation, must be repeated multiple times – at least 50 repetitions – to obtain stable results.

The figure also shows that LSM–CRRA provides a high-accuracy estimate, with confidence bands indicating that deviations from the exact value remain within 5%.

Figure 3b, shows that MBRT–CRRA and LSM–CRRA converge across different values of γ. For positive coefficients (e.g. +0.8) convergence is reached quickly due to the stability of



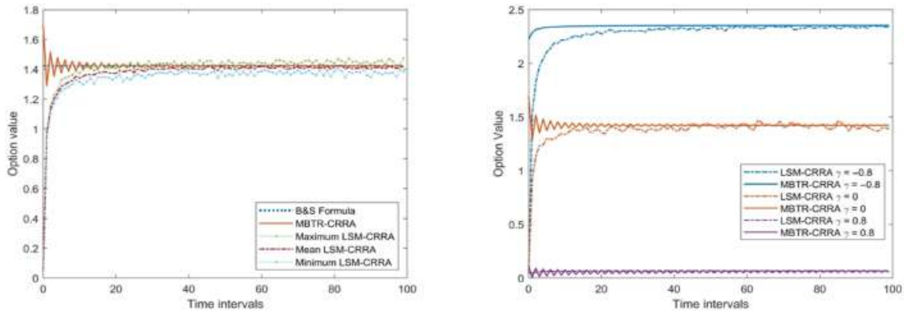
Behavior of S_t vs Y_t respect γ –0.2, 0 and +0.2

(a)

Behavior of Y_t respect γ $-0.9 \leq \gamma \geq 0.9$

(b)

Figure 2. Behavior of Yt respect to γ with N = 250. **Source(s):** Authors' own elaboration



(a) Comparison of Black-Scholes method, MBRT-CRRA and LSMCRRA (with min-max intervals) schemas to value a European Call option

(b) Comparison of Black-Scholes, MBRT-CRRA and LSM-CRRA for RRA values of −0.8, 0 and 0.8 to value an European Call option, under the initial conditions of $S_0 = 10$, $K = 10$, $N = 100$, $\sigma = 0.3$, $r = 0.05$ and $T = 1$ years.

Figure 3. Comparison of Black–Scholes method, MBRT–CRRA and LSM–CRRA. **Source(s):** Authors' own elaboration

LSM–CRRA relative to the dispersion of the MBRT–CRRA. When γ is negative, the convergence between the methods occurs near the end of the proposed interval. In this scenario, the MBRT–CRRA method proves to be more stable while the LSM–CRRA tends to exhibit higher volatility, particularly with a coefficient of 0.8. In both cases, the exact value cannot be derived using B-S, since it lacks risk-adjustment terms, underscoring the motivation for our framework and the scope for deriving an exact differential equation in future research.

4.4.1 Comparison of MBRT–CRRA and LSM–CRRA. The primary objective of this study is to present a numerical framework that incorporates risk aversion into option valuation. To this end, we conducted four sensitivity analyses to examine how the MBRT–CRRA and LSM–CRRA methods capture investor risk preferences and their impact on option premiums. These analyses were performed with $S_0 = 10$, $K = 10$, $r = 0.05$, annualized volatility of $\sigma = 0.30$, $T = 1$, $N = 300$, and using the LSM–CRRA method with four regressors.

Figure 4 clearly illustrates how the variation in annualized volatility (σ) and the γ coefficient affect option prices across European, American and Bermudan (call and put options) under the LSM–CRRA and MBRT–CRRA frameworks. The sensitivity analysis demonstrates a pronounced asymmetry: when investors are risk-loving, they consistently assign higher values to options as volatility increases. Conversely, for risk-averse investors, option prices initially decrease with low volatility and only begin to rise again at higher volatility levels. For example, at $\gamma = +0.1$, option values initially decline within the 1%–3% volatility range but subsequently increase as volatility rises beyond this threshold. As γ approaches higher levels of risk aversion (e.g. $\gamma = +0.7$), this recovery of option values is delayed, indicating a stronger aversion to volatility.

These patterns are further detailed quantitatively in Table 3, which compares the mean values produced by MBRT–CRRA and LSM–CRRA for Americans put under various volatility and risk preference scenarios. Using the risk-neutral scenario $(\gamma = 0)$ as a baseline,

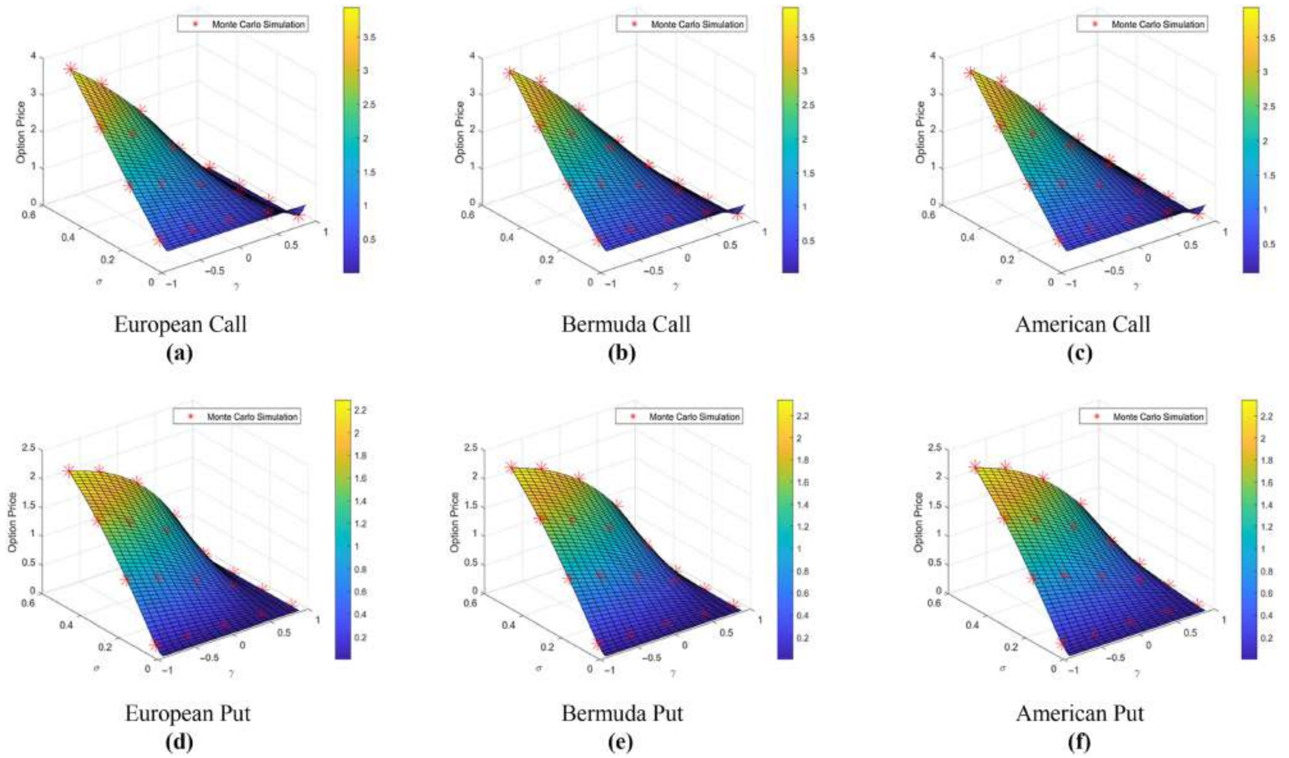


Figure 4. Comparison between the means of LSM–CRRA and MBRT–CRRA through an analysis of sensitivity to volatility. **Source(s):** Authors' own elaboration

Table 3. Comparison of the value of a Bermudan call option between the MBRT–CRRA and the LSM–CRRA

Sigma// gamma	−0.9 min	mean	max	−0.5 min	mean	max	0 min	mean	max	0.5 min	mean	max	0.9 min	mean	max
0.05	0.14*			0.120*			0.082*			0.04*			0.006*		
	0.141	0.152	0.158	0.117	0.122	0.126	0.08	0.083	0.085	0.04	0.042	0.044	0.007	0.008	0.009
0.2	0.982*			0.831*			0.608*			0.338*			0.074*		
	0.964	0.995	1.03	0.824	0.846	0.876	0.599	0.617	0.645	0.315	0.342	0.368	0.064	0.074	0.082
0.35	1.732*			1.524*			1.175*			0.691*			0.15 *		
	1.7	1.749	1.815	1.51	1.544	1.601	1.153	1.192	1.228	0.663	0.699	0.737	0.148	0.16	0.176
0.5	2.339*			2.139*			1.742*			1.086*			0.252*		
	2.291	2.348	2.411	2.113	2.165	2.227	1.695	1.753	1.803	1.047	1.1	1.144	0.241	0.266	0.292

Note(s): In each volatility scenario, the first row displays the MBRT–CRRA premium (*), while the second row presents the minimum, mean and maximum values of the LSM–CRRA method over 50 repetitions

Source(s): Authors' own elaboration

we observe consistently lower premiums compared to risk-loving scenarios, underscoring the economic intuition behind investor preferences. Specifically, risk-loving investors perceive high volatility positively, as it enhances the probability of substantial gains while limiting losses to the premium paid. Consequently, they assign higher premiums to options. Conversely, risk-averse investors emphasize potential downside risk by discounting option premiums, especially at higher volatility levels.

Overall, these results confirm that the MBRT–CRRA framework effectively captures how risk preferences shape the perceived value of optionality. Risk-loving investors, embracing uncertainty, systematically value options more highly, a behavior clearly highlighted both visually and quantitatively in Figure 4 and Table 3.

Figure 5 illustrates how variations in the risk-free rate and the RRA coefficient affect option premiums within the LSM–CRRA and MBRT–CRRA frameworks. This sensitivity analysis reveals a clear interaction between investor preferences and changes in the risk-free rate. Specifically, option values systematically decrease as investors become more risk-averse (positive γ), whereas they increase substantially under conditions of risk-loving behavior (negative γ). Higher risk-free rates further amplify these patterns, notably increasing option premiums for all investor types but especially benefiting risk-loving investors.

These findings are quantitatively confirmed in Table 4, which presents numerical comparisons between MBRT–CRRA and LSM–CRRA valuations for European call at various levels of the risk-free rate and γ. Using the risk-neutral scenario $(\gamma = 0)$ as a reference, premiums consistently decrease as risk aversion increases $(\gamma > 0)$, and significantly increase under risk-seeking attitudes $(\gamma < 0)$. For instance, at $r = 0.5$, the premium with $\gamma = -0.9$ (strongly risk-loving) reaches to 4.253, while for $\gamma = +0.9$ (strongly risk-averse), it sharply declines to 3.239.

The economic intuition behind these behaviors lies in the way different investors value uncertain future payoffs. Risk-loving investors perceive an increase in the risk-free rate as an opportunity to magnify potential gains without heavily discounting potential losses, making the optionality extremely valuable. Conversely, risk-averse investors remain cautious, heavily discounting uncertain outcomes, thereby lowering the perceived value of options even when risk-free rates rise.

Figure 6 illustrates how variations in option maturity and the γ coefficient influence option prices. Risk-loving investors consistently assign higher premiums as maturity increases, reflecting their preference for the higher uncertainty associated with longer time horizons. In contrast, risk-averse investors value options much less, even at extended maturities, due to heightened sensitivity to potential losses.

Table 5 quantitatively confirms these patterns for Bermudan puts, highlighting that at longer maturities (e.g.$T = 2$), option premiums are significantly higher for risk-loving investors (1.202 at $\gamma = -0.9$) compared to strongly risk-averse investors (0.008 at $\gamma = +0.9$). Thus, the MBRT–CRRA framework effectively captures how investor attitudes toward risk and time shape the perceived value of optionality.

Figure 7 demonstrates how variations in strike prices (K) and the RRA coefficient influence option premiums. Notably, risk-loving investors place substantially higher premiums on deep out-of-the-money options, reflecting their preference for risky bets with high payoffs despite lower probabilities of success. Conversely, risk-averse investors significantly reduce premiums as options move further out-of-the-money, emphasizing their aversion to unlikely favorable outcomes.

Table 6 quantitatively confirms these insights for American call: at lower strikes (e.g. $K = 5$), premiums remain high for all investors but increase substantially for risk-seeking investors ($\gamma = -0.9$, with 5.479 as premium) compared to risk-averse ones ($\gamma = +0.9$, with 5.000 as premium). However, as strikes rise (e.g. $K = 15$), this difference becomes even more pronounced, with premiums nearing zero for strongly risk-averse investors, reflecting their unwillingness to pay for highly uncertain payoffs. Thus, investor risk attitudes significantly shape how strike variations affect perceived option value, which is clearly captured by the MBRT–CRRA framework.

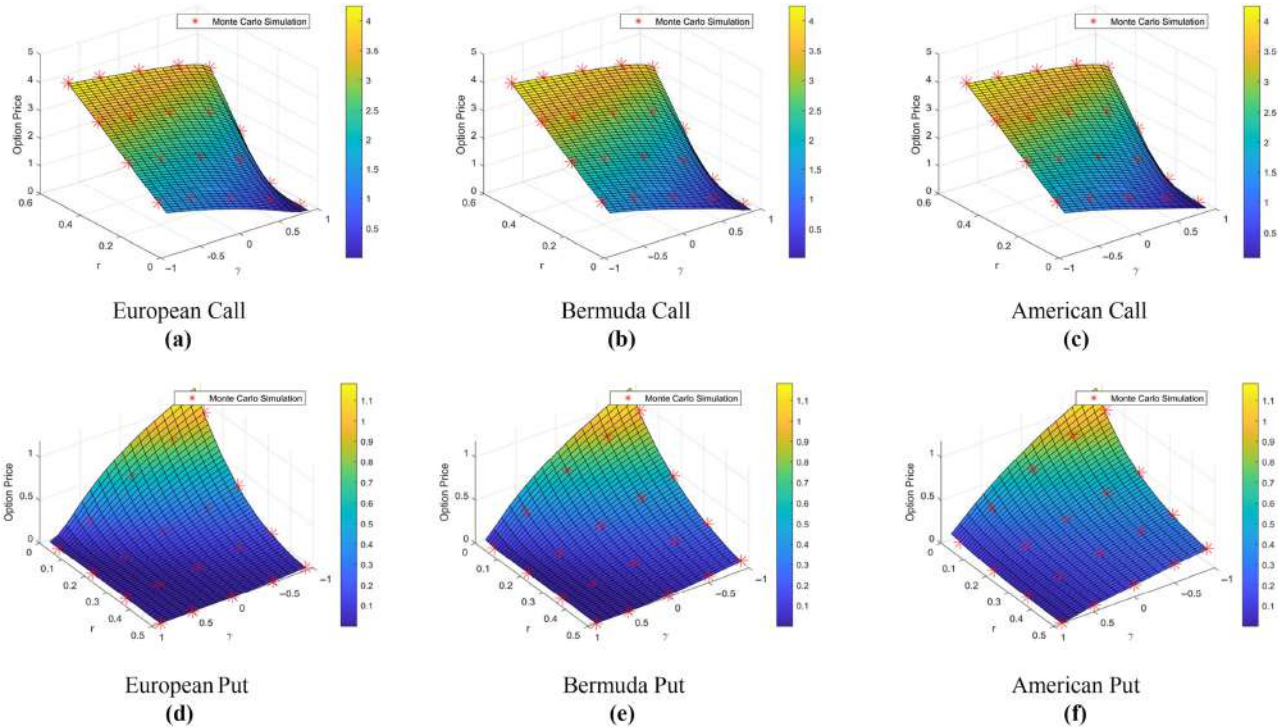


Figure 5. Comparison between the means of LSM–CRRA and MBRT–CRRA through an analysis of sensitivity to the risk-free rate and CRRA coefficient. **Source(s):** Authors' own elaboration

Table 4. Comparison of MBRT–CRRA and LSM–CRRA to value a European Call γ vs r

r//	−0.9			−0.5			0			0.5			0.9		
gamma	min	mean	max	min	mean	max	min	mean	max	min	mean	max	min	mean	max
0.05	1.698*			1.437*			1.040*			0.49*			0.00*		
	1.626	1.704	1.773	1.396	1.452	1.497	1.008	1.045	1.097	0.453	0.493	0.542	0.003	0.004	0.005
0.2	2.546*			2.311*			1.947*			1.390*			0.237*		
	2.501	2.565	2.642	2.26	2.327	2.37	1.907	1.958	2.009	1.289	1.398	1.455	0.197	0.235	0.288
0.35	3.422*			3.234*			2.955*			2.560*			1.556*		
	3.385	3.457	3.526	3.203	3.261	3.319	2.904	2.974	3.031	2.478	2.573	2.625	1.401	1.527	1.645
0.5	4.253*			4.109*			3.907*			3.659*			3.249*		
	4.191	4.289	4.349	4.077	4.139	4.205	3.892	3.935	3.998	3.609	3.672	3.764	3.134	3.236	3.31

Note(s): In each risk-free scenario, the first row shows the MBRT–CRRA premium (*), while the second row reports the minimum, mean and maximum values of the LSM–CRRA method for 50 repetitions

Source(s): Authors' own elaboration

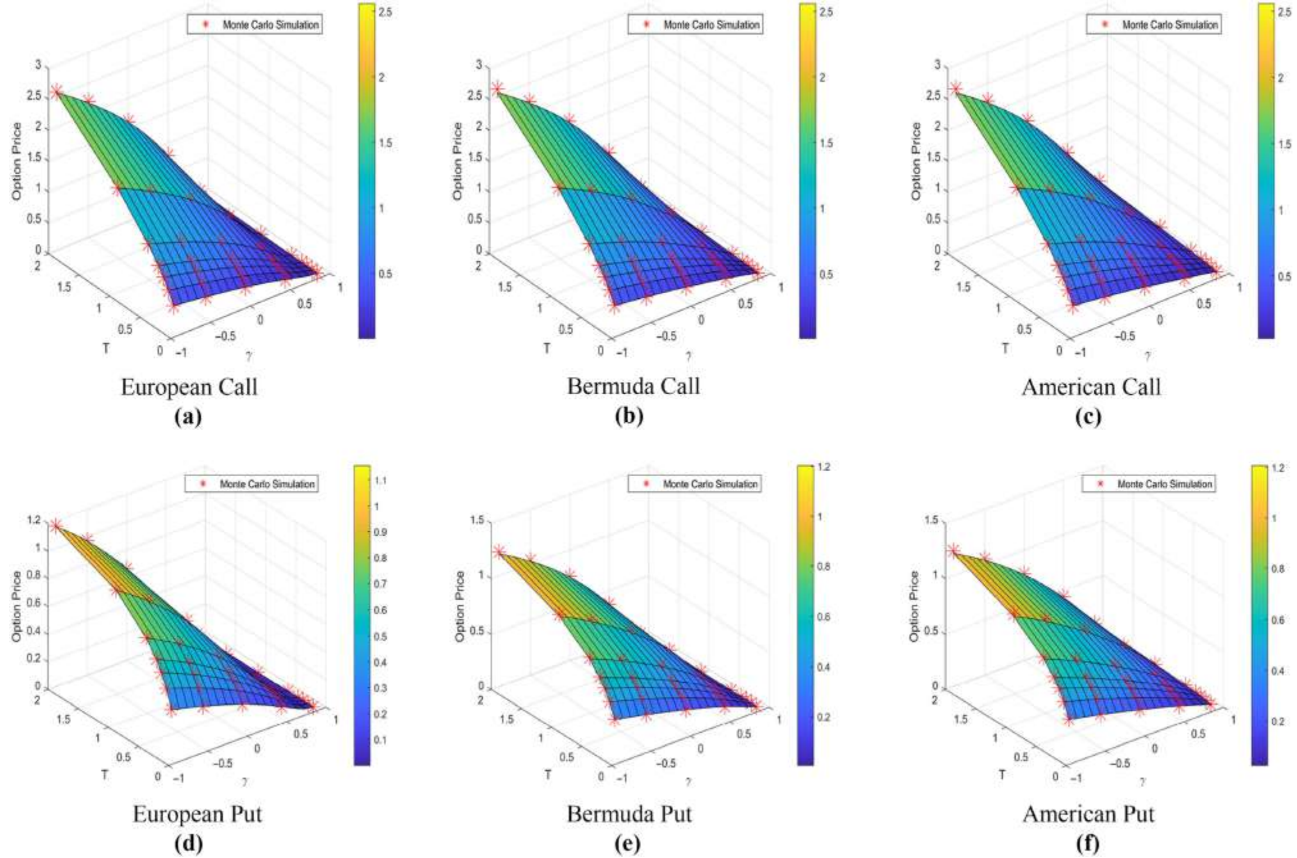


Figure 6. Comparison between the mean of LSM–CRRA and MBRT–CRRA schemes through a sensibility analysis with respect to expiration maturity and γ CRRA coefficient.
Source(s): Authors' own elaboration

Table 5. Comparison of MBRT–CRRA and LSM–CRRA to value a Bermudan Put γ vs T

T//	−0.9			−0.5			0			0.5			0.9		
gamma	min	mean	max	min	mean	max	min	mean	max	min	mean	max	min	mean	max
1//12		0.360*			0.300*			0.21*			0.10*			0.00*	
	0.354	0.363	0.373	0.294	0.304	0.315	0.206	0.215	0.222	0.104	0.11	0.117	0.004	0.005	0.005
1//6		0.490*			0.40*			0.290*			0.14*			0.005*	
	0.479	0.495	0.514	0.398	0.412	0.43	0.279	0.295	0.307	0.142	0.149	0.158	0.004	0.006	0.008
1//4		0.582*			0.486*			0.34*			0.17*			0.006*	
	0.567	0.589	0.608	0.475	0.492	0.509	0.338	0.351	0.365	0.165	0.177	0.186	0.006	0.007	0.009
1//3		0.655*			0.548*			0.39*			0.199*			0.007*	
	0.64	0.662	0.682	0.538	0.556	0.581	0.381	0.396	0.411	0.189	0.2	0.213	0.005	0.007	0.009
1//2		0.768*			0.64*			0.463*			0.234*			0.007*	
	0.75	0.775	0.799	0.63	0.653	0.671	0.442	0.465	0.48	0.217	0.236	0.254	0.006	0.008	0.01
1		0.982*			0.829*			0.601*			0.302*			0.008*	
	0.956	0.993	1.027	0.802	0.839	0.875	0.579	0.605	0.632	0.286	0.302	0.319	0.007	0.009	0.011
2		1.202*			1.026*			0.750*			0.371*			0.008*	
	1.171	1.208	1.241	1.006	1.036	1.069	0.723	0.754	0.799	0.351	0.373	0.396	0.007	0.009	0.011

Note(s): In each expiration scenario, the first row displays the MBRT–CRRA premium (*), while the second row presents the minimum, mean and maximum values of the LSM–CRRA method for 50 repetitions

Source(s): Authors' own elaboration

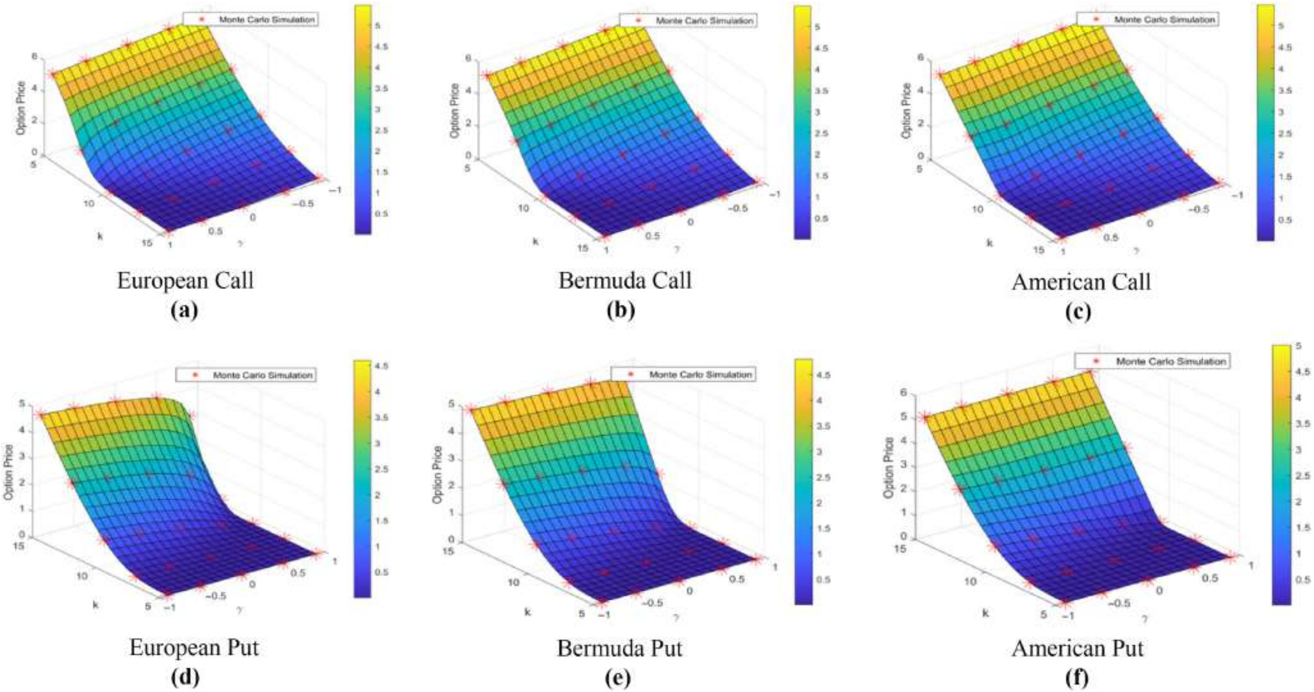


Figure 7. Comparison between the mean of LSM–CRRA and MBRT–CRRA schemes through an analysis of sensitivity to strikes and γ RRA coefficient. **Source(s):** Authors' own elaboration

Table 6. Comparison of MBRT–CRRA and LSM–CRRA to value an American call γ vs K

K//	−0.9			−0.5			0			0.5			0.9		
gamma	min	Mean	max	min	Mean	max	min	Mean	max	min	Mean	max	min	Mean	max
5		5.479*			5.377*			5.242*			5.095*			5.000*	
	5.492	5.513	5.553	5.336	5.39	5.445	5.214	5.264	5.328	5.032	5.124	5.198	5.013	5.046	5.072
7.5		3.379*			3.184*			2.893*			2.523*			2.500*	
	3.382	3.405	3.446	3.187	3.201	3.223	2.839	2.902	2.96	2.513	2.552	2.584	2.498	2.503	2.514
10		1.698*			1.437*			1.040*			0.529*			0.105*	
	1.626	1.706	1.78	1.445	1.466	1.502	0.985	1.047	1.079	0.517	0.538	0.567	0.102	0.11	0.123
12.5		0.692*			0.485*			0.227*			0.034*			0.000*	
	0.715	0.731	0.757	0.45	0.494	0.536	0.22	0.235	0.258	0.033	0.037	0.04	0	0	0
15		0.240*			0.131*			0.034*			0.000*			0.000*	
	0.237	0.262	0.3	0.136	0.15	0.177	0.025	0.038	0.049	0.001	0.001	0.002	0	0	0

Note(s): In each case of strike value, the first row shows the value of the MBRT–CRRA premium (*), while the second row reports the minimum, mean and maximum values obtained from the LSM–CRRA method across 50 repetitions

Source(s): Authors' own elaboration

5. Discussions

5.1 Theoretical and practical implications

The integration of CRRA utility functions into derivative valuation represents a key innovation, as it explicitly accounts for investor risk preferences beyond the traditional risk-neutral paradigm. This framework advances the literature on utility-based and nonlinear risk-adjusted pricing by demonstrating how variations in RRA coefficient shape option values.

From a practical perspective, the model offers a flexible tool for portfolio managers, traders, and policymakers by aligning valuations with observed market behavior while preserving consistency with classical models under specific conditions. Accurate calibration of risk preferences thus becomes essential for reliable pricing and policy design.

5.2 Limitations

While this research offers significant theoretical and practical contributions, certain limitations warrant further study.

(1) Computational complexity: The CRRA-based valuation method introduces additional numerical challenges, requiring precise calibration and advanced optimization techniques.

(2) Dependence on risk aversion estimates: The model assumes that investor risk preferences can be accurately measured and remain stable, which may not always reflect dynamic market conditions.

(3) Absence of a closed-form solution: Our proposed approach lacks an analytical closed-form solution. Therefore, it is crucial to dedicate efforts to its development.

5.3 Future research agenda

To enhance the model's applicability and theoretical foundation, future research should focus on.

(1) Epstein–Zin recursive utility – Integrating Epstein–Zin preferences to separate risk aversion from intertemporal elasticity of substitution, refining the modeling of investor behavior.

(2) Machine learning techniques – Leveraging neural networks and reinforcement learning to optimize parameter estimation and computational efficiency.

(3) Empirical validation with market data – Calibrating the RRA coefficient using real-world option market data to assess the model's pricing accuracy compared with traditional approaches.

(4) Closed-form solutions – Developing approximate analytical solutions to address the lack of a closed-form expression in CRRA-based derivative pricing.

6. Conclusions

This study introduces a numerical framework for option pricing that incorporates investor risk aversion through a CRRA utility function, enabling option values to reflect individual risk preferences via the RRA coefficient. Unlike traditional risk-neutral methods, this approach offers a more realistic and flexible valuation tool.

Key results show that variations in the RRA coefficient γ significantly affect option values through adjustments to the modified risk-free rate α, revealing a non-monotonic relationship between risk aversion and pricing. This underscores the importance of accurate calibration of risk preferences for robust valuation outcomes.

The framework is applicable to American, European and Bermudan call and put options, subject to mathematical conditions of constrained probability, positivity and monotonicity, and provides a foundation for future research on convergence and closed-form solutions. Overall, the contribution lies in expanding asset pricing theory by embedding investor-specific risk preferences into derivative valuation, enhancing both theoretical and practical applications, and fostering a deeper understanding of risk-adjusted option pricing in dynamic financial environments.

Notes

1. In this work, we adopt the Itô interpretation of the stochastic integral, which is the standard in financial modeling and aligns with risk-neutral pricing theory. However, in systems with multiplicative noise, alternative interpretations – such as Stratonovich or Klimontovich–Hänggi – lead to different volatility corrections and thus distinct probability density functions and moment behaviors. This nuance has been extensively discussed in the physics literature (see Mannella and McClintock, 2012) and more recently in mathematical treatments (Escudero and Rojas, 2023). While these frameworks are beyond the scope of the present work, we acknowledge their relevance and suggest they be explored in future research on option pricing under non-standard noise conventions.
2. In this derivation, we apply Itô's lemma, which generalizes the chain rule for stochastic processes, where $f \in C^2$Type equation here. Specifically, for a function $Y_t = f(S_t)$, Itô's formula is: $df(S_t) = f'(S_t)dS_t + \frac{1}{2}f''(S_t)(dS_t)^2$Type equation here. This formulation corresponds to the Itô interpretation of stochastic integration, which differs from Stratonovich and Klimontovich–Hänggi interpretations often used in physics. The Itô interpretation is standard in financial modeling due to its compatibility with martingale measures and arbitrage-free pricing.
3. Beyond the classical GBM framework, these moment results have also been generalized to more complex stochastic processes, such as continuous-time random walk and fractional models. While our focus here is on recombination schemes consistent with GBM, acknowledging these extensions highlights the broader applicability of moment-matching methodologies.
4. In our LSM–CRRA implementation we follow the classic Longstaff–Schwartz algorithm, mapping the underlying price St to the CRRA-adjusted state variable Yt, so that risk preferences enter only through the CRRA transformation while the valuation itself remains risk-neutral. For every exercise date, we retain the in-the-money paths and regress their discounted cash flows $\widehat{Y}$ onto a compact set of low-order polynomials in the normalized price, specifically Legendre-type polynomials. The regression is a plain OLS estimation, repeated backwards in time, projecting and comparing the continuation value with the immediate exercise payoff, so that payoffs are overwritten only when early exercise is optimal. For a detailed description of the full method and algorithmic steps, the reader is referred to Section 3.4 and 3.5 of Marín-Sánchez *et al*. (2025), where the complete framework and pseudocode are provided. This material is not reproduced here due to space constraints.

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